

PIAGET'S
CONSTRUCTIONIST EPISTEMOLOGY

by

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AN EXPLORATION AND A COMPARISON
WITH SEVERAL ALTERNATIVE THEORIES

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PREFACE

Visitors to the Heuristics Laboratory frequently ask one of two questions: "What does Piaget have to do with teaching college level physics?" or "What can I read that is a good introduction to Piaget?" These questions can be answered in several different ways.

On one level, Piaget's theory describes a series of intellectual stages which must be passed through in order to master the kinds of formal reasoning processes necessary for an understanding of physics. This aspect of the theory provides us with both a description of the stages, and a battery of tests for determining an individual's current state of intellectual development. To a physicist, these tests carry an inherent validity because they are in fact tests of knowledge about the physical world. Thus, part of the reason for the popularity of Piaget among physicists is that they have not had to concern themselves with whether the stages accurately describe intellectual growth in general. It is sufficient that the theory describe the particular type of growth necessary for physics. Two good introductory references that describe the stages are:

H. Ginsburg and S. Opper. Piaget's Theory of Intellectual Development.

P.G. Richmond. An Introduction to Piaget.

It is perhaps unfortunate that the stage aspects of Piaget's theory are so easy to accept since this tends to distract attention from the basic structural foundations of the theory. Piaget's major contribution

to the field of cognitive psychology lies in his successful application of a constructionist philosophy of knowledge to the study of human intellectual development. This aspect of the theory is difficult to understand. Furthermore, prior to reading Herb Koplowitz' paper, I knew of no suitable introduction to the subject.

Here, then, is a succinct treatment of Piaget's complex theory with special emphasis on clarifying the distinction between it and other psychological theories. It is followed by a shorter paper outlining some of the implications of this theory for education.

Piagetian psychology has its own jargon. Please pay attention to the specialized meanings attached to the following words:

structures of knowledge
scheme
assimilation
accommodation
equilibration
constructionist epistemology

Readers interested in a short introduction may wish to skip the section on Alternative Theories. I would, however, recommend reading the section on Skinner, pages 33 - 36. Those interested in additional readings are referred to:

S. Kuhn. The Structure of Scientific Revolutions.

R.M. Pirsig. Zen and the Art of Motorcycle Maintenance.

Jack Lochhead

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INTRODUCTION

The world presented to our senses is essentially continuous in space and time. We experience the world, however, as consisting of objects and events which may be bounded in space and time. In knowing the world, we break it up and structure it.

The simplest observations of children show that we are not born with our ways of breaking up the world. A young child may not distinguish between dogs and cats, and will not even judge number or length or speed as adults do (Piaget, 1946, 1952). The structures of our knowledge must be developed, and the major issue this paper focusses on is how those structures develop. In particular, the paper will center on Piaget's position on this issue and how it compares to several alternative positions.

The alternative positions discussed here hold in common that the structures of knowledge are copies of other structures. B.F. Skinner and Eleanor Gibson hold that the structures of knowledge are copies of the structures of the world. Benjamin Whorf and the logical positivists held the structures of knowledge to be taken from the structures of language. The gestaltists tried to reduce the structures of knowledge to the structures of perception.

Piaget, on the other hand, believes the structures of knowledge to be constructed rather than copied. To explicate the notion of construction, a brief description of Piaget's theory will be given shortly. This discussion should also clarify the notion of a "structure of knowledge" which is more central to Piaget's theory than to some of the others which will be discussed.

After Piaget's theory is introduced, and before alternative theories are discussed, the importance of this paper's topic should be established: why would anyone be interested in the structures of knowledge and how they develop? This discussion should lay the grounds for the analysis of the alternative theories.

One purpose of discussing the alternative theories is to highlight their weaknesses. Some of them have a common-sense appeal that makes them attractive, especially in comparison to Piaget's notoriously difficult theory. An exposition of the weaknesses of other theories may make it easier to understand why Piaget would propose his own. In the final section of the paper, Piaget's theory will be discussed in greater depth. Attention will be given to the question of how well his theory avoids the weaknesses of the alternative theories, and the weaknesses of his own theory will be discussed.

Piaget's Theory. Schemes: Central to the theory is the notion of a "scheme", which may be best approximated as being a synonym for "concept". A person is not necessarily aware of his own schemes, and certainly does not usually employ them consciously, but rather exhibits his schemes in his actions.

For example, an infant may have a scheme for sticking his finger into holes in objects. There will be a regularity in the way he discovers the hole, sticks his finger in it, retrieves his finger, and perhaps repeats the last two steps. This regularity is what indicates the presence of the scheme: it is evident that the infant is doing the same thing each time he is presented with an object with a hole in it.

For another example, one which Piaget has examined in depth, we may consider a person's scheme for judging amount. The differences between the five-year-old's scheme for amount and the eight-year-old's scheme are highlighted by having the children make judgments about the amount of clay contained in various lumps. Piaget (1956) has shown that the younger child will think the amount in a lump has changed when it is rolled into a string, while the older child will think the amount has not changed. The children have different schemes for amount.

Furth (1969, p. 264) defines "scheme" as "The internal general form of a specific knowing activity . . . The generalizable aspect of coordinating actions that can be applied to

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analogous situations." In fact, it is a person's schemes which determine when two situations are analogous. It is only because of his sticking-the-finger-in-the-hole scheme that the infant considers holding a cup to be similar to holding a light socket. And in the example of the five-year-old and the eight-year-old, only the latter considers the situation of two equal lumps to be the same after one has been rolled out. The schemes determine when two situations are the same, and this will allow the same action to be performed on them. In the example of the infant, the "same action" is simply sticking a finger in the hole. In the other example, the "same action" is, for the older child, calling the two lumps equal and treating them as equal.

Assimilation: Piaget takes the notion of assimilation, "taking into a structure", from biology. To assimilate something means very nearly the same thing as to respond to it. The important difference is that in assimilating something, one also defines it, and the definition is in terms of the assimilator's schemes.

Our infant assimilates a cup as "an object to put a finger into". He can do this only because he has the appropriate scheme. A younger infant might only be able to grab the cup, and would thus assimilate the cup as "something to grab". Our children assimilated the lumps of clay to different schemes for amount, and thus produced different judgments and different behaviors.

In Piaget's theory, to know something is to assimilate it to a scheme. Thus, a person's schemes are the structures of his knowledge. Our infant knows the cup as something to put a finger into. A younger infant could only know it as something to grab. Our five-year-old knows the lump of clay and the string of clay to be unequal while the eight-year-old knows them to be equal. Our schemes determine what we know things as, and all other judgments we make.

Accommodation: When a scheme is used, it may need to be changed in order to fit the particulars of a new situation. This change is called accommodation. Accommodation may be simply the stretching of a general scheme to fit the particulars of a given instance, or it may entail creation of a new scheme.

If we give our infant cups of increasing size, he will have to change slightly the way in which his finger explores the inside of the cup before withdrawal. By the time the containers reach pail size, a different action may be called for. If the infant's behavior indicates a distinctly different approach toward small cups and larger containers, we may say that the accommodation has led to creation of a new scheme. As Furth (1959, p. 229) has said, however, "There are no a priori rules for judging the extent of modification requisite for calling the result a new scheme."

Splitting in two is just one way in which accommodation can lead to new schemes. Two schemes can also join to form a new one. At some point in his development, every infant combines his scheme for grasping with his scheme for watching objects so he can systematically watch what he grabs and can grab what he is watching.

Another form of accommodation is restructuring. For example, a child usually first judges volume according to the largest dimension. (The longer piece of clay is larger.) Later, he will judge it according to the smallest dimension. (The thinnest is smallest.) Later, he will vacillate between the two. Finally, he will integrate dimensions and attain an adult concept of volume. Each change in the volume scheme is a restructuring.

The concepts of accommodation and assimilation are linked so that it is sometimes difficult to distinguish them. Roughly, assimilation is the modification of observations to make those observations fit internal models, (schemes). ~~Accommodation~~ is the modification of internal models to make them fit observations.

Factors influencing development of schemes: Accommodation can only take place when something is assimilated to a scheme, and generally this happens only in the presence of something assimilable to that scheme. Thus, the environment is the first factor cited by piaget as affecting

development of schemes. (When discussing these factors, Piaget usually separates this first one into physical environment and social environment. The distinction is not relevant to the present discussion).

The infant's schemes relating to cups can only be changed if he has cups to play with. Schemes relating to amount and volume can only change as a result of experience in judging amount and volume of physical things. Note that environment does not play the same role in Piaget's theory as it does in Skinner's (which is discussed in greater detail below.)" The environment does not act on the person, eliciting responses and affecting the probabilities of the occurrence of responses. Rather, the person acts on the environment; the environment affects development by providing situations assimilable to some schemes but not to others.

The second factor affecting development of schemes is maturation. Piaget is vague about how maturation affects development, but he does not see it as a process of certain schemes simply appearing after a person attains the correct age. Rather, he seems to see maturation as a limiting factor. Certain schemes need a relatively advanced state of neurological development before they can appear. Thus, maturation does not cause a scheme to appear, but only allows it to develop.

The third factor in development is equilibration. Furth (1969) describes equilibration as follows:

8.

It is conceived as the factor that internally structures the developing intelligence. It provides the self-regulation by which intelligence develops in adapting to external and internal changes. . . . Piaget insists on the factor of equilibration partly because he needs a unifying principle of development and cannot accept other factors as that principle. (pp. 206-207).

As I understand equilibration, it is the factor which "notices" self-contradiction within a person's knowledge and which reduces this imbalance through construction of new schemes. The function of equilibration is much clearer than its operation. That is, Piaget tells us more about what equilibration does than about how it does it. I will return to this point at the end of the paper.

Knowledge as construction: Piaget's epistemology is constructionist in two ways. First, the notion of assimilation entails construction. By use of this notion, Piaget emphasizes his position that cups are not given to us as cups, and that equality does not present itself as such, but that we know cups and equalities through our schemes; we make that object a cup, we judge those lumps to be equal, and all of our knowing consists of such constructive actions.

More importantly for this paper, accommodation entails construction. When a new scheme is developed, it is not copied from somewhere, and the new scheme is not given in the old scheme or in new data. We might say the new scheme is created through equilibration. Again, I will return to the adequacy of this account of the formation of new schemes later in the paper. What should be understood at this point,

however, is that Piaget sees the structures of knowledge as being constructed by the knower as a result of his own actions rather than the structures being derived from some external source. Each structure is constructed from earlier structures and all eventually traceable to a few "prewired" structures such as the **sucking scheme**.

ON STUDYING THE STRUCTURES OF KNOWLEDGE
AND THEIR FORMATION

Why Study the Structures of Knowledge

Not all who have studied knowledge have focussed on the structures of knowledge. It remains to be explained why Piaget's theory, and this paper, would center on those structures. We will first consider the role structures of knowledge play in Piaget's theory. Then, two other attitudes towards the structures of knowledge will be discussed: Skinner's view that there is no need to study the structures of knowledge, and the view implicit in some cognitive psychologists' studies that assumptions about structures of knowledge need not be tested empirically nor examined philosophically.

Piaget's Position

Systematic errors in knowledge: One of Piaget's tasks is to explain how it is possible for us to have valid knowledge.¹ He begins with an emphasis on systematic errors in knowledge, and attempts to explain how a person's knowledge becomes more valid. Some examples of these systematic errors are the following:

1. Bransford and Franks (1971) presented subjects with sentences based on four simple ideas, for example: "The ants

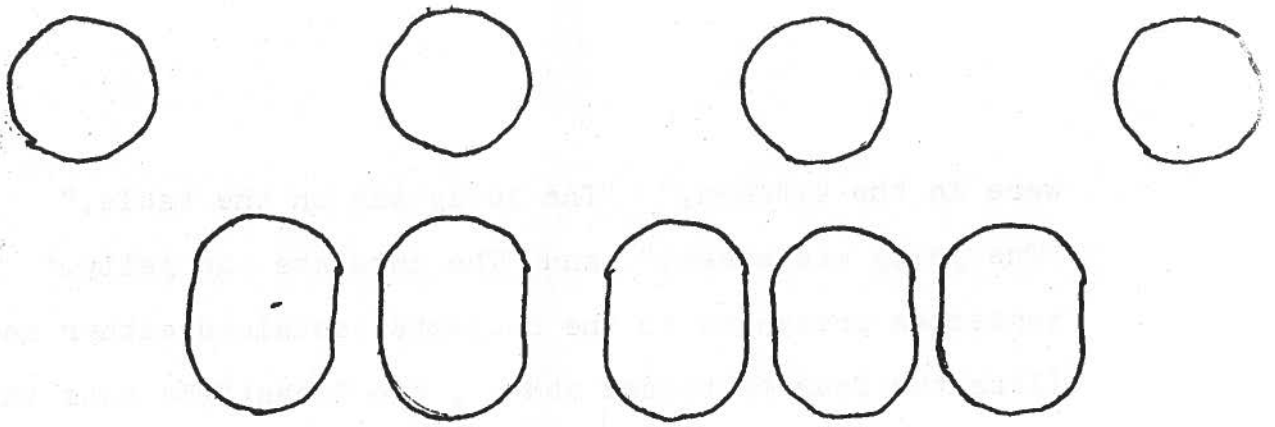


Figure 1

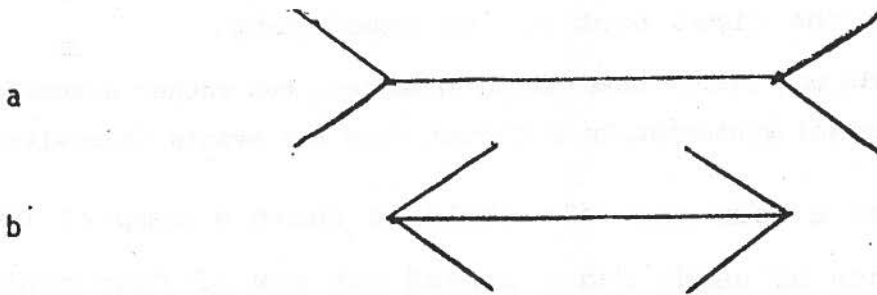


Figure 2

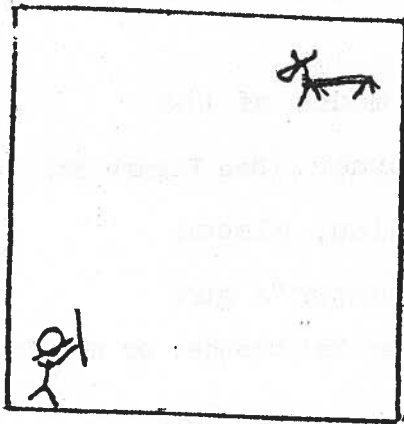
were in the kitchen," "The jelly was on the table," "The jelly was sweet," and "The ants ate the jelly." The sentences presented to the subjects contained either one idea (like the four sentences above), two ideas ("The ants in the kitchen ate the jelly"), or three ideas ("The ants ate the sweet jelly which was on the table"). In the recognition part of the experiment, subjects were presented with sentences they had heard and sentences they had not heard. They were asked to specify for each sentence whether it had been presented earlier, and how sure the subject was of his judgment. Subjects tended to judge that they had heard four idea sentences ("The ants in the kitchen ate the sweet jelly which was on the table") and were surer of these judgments than of any others they made. In fact, no four idea sentences had been presented in the first part of the experiment.

The subjects did not simply have faulty memories, but rather a knowledge of events which was systemically different from the events themselves.

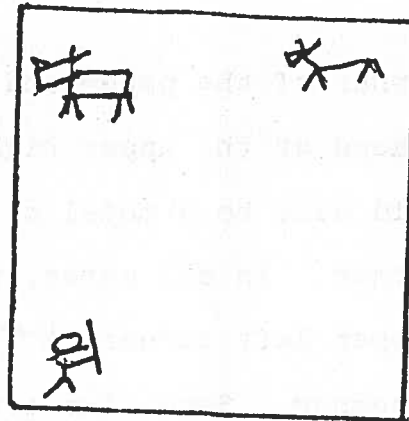
2. When a four year old child is shown a compact row of five pieces of candy and a spread out row of four pennies, he knows there are more pennies than candies. (See Figure 1.)

3. When a naive subject looks at the Müller-Lyer figure, shown in Figure 2, he judges the horizontal line in 2a to be longer than that in 2b even though the two are the same length.

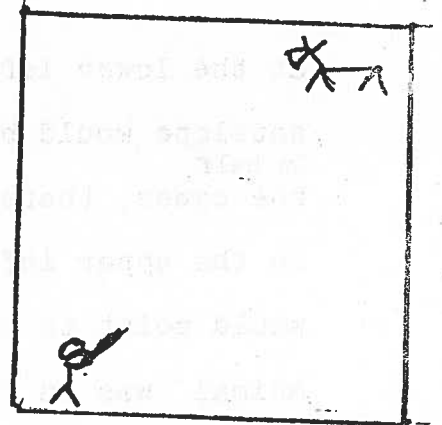
4. Michael Cole (1973) reported an experiment in which Africans were shown models of animals and of a hunter placed on a piece of paper. The model of the hunter would be placed



a



b



c

Figure 3

at the lower left corner of the paper and the model of the antelope would be placed at the upper right corner. (See Figure 3a). In half the cases, there would also be a model of a rhino, placed in the upper left corner. In all cases, the hunter's gun would point to the upper left corner, (see Figure 3b) whether or not an animal was in that corner. Some time after presentation, the subjects were given the models that they had seen previously and were asked to place them on the paper as they had seen them. Those who had been shown the scene with three models placed the models correctly, with the hunter in the lower left aiming at the rhino in the upper left, who had been shown the scene without the rhino tended to place the antelope, correctly, in the upper right corner, and the hunter in the lower left corner with his gun incorrectly pointing toward the antelope, as in Figure 3c.

Knowledge as copy vs. knowledge as construct: These examples indicate not that we are sometimes mistaken in our beliefs, but that we are systematically mistaken; the errors are not random. What this suggests to Piaget is that knowledge is not a copy of reality but a transformation or a construction of reality.

Knowledge is a transformation of reality in that by knowing things we attribute to them properties they do not really have.

We know we have heard sentences which in fact we have

not heard. We know equal lines to be unequal. We know the hunter was aiming at the antelope when he was actually aiming at nothing. When we were children, we knew larger amounts to be smaller. Obviously, our knowledge is not maladaptively invalid: we do not change the world so much that we constantly endanger ourselves. But it would seem an error to think of knowing as being the passive recording of events.

Knowledge is a construction in that we build our knowledge from our sensory input; again, the input and the knowledge are not identical. Consider the subject who was shown the models of the hunter, antelope, and rhino, who correctly aimed the hunter's gun at the rhino. It would be tempting to look at this subject's resetting of the figures as playing back a recording of the scene originally shown him. It would seem, however, that the subject made use of his knowledge of guns as things which are pointed at game, as well as his knowledge of animals as things which are shot. In other words, this subject was constructing the scene so it would make sense to him just as other subjects were incorrectly constructing the scene without the rhino in a way that made that scene more cogent to them.

One reason Piaget is interested in structures of knowledge is that he finds them useful in describing the constructive aspect of knowledge, which he does using the mathematical notion of a function.

Piaget's function from reality to knowledge: A function assigns to each element of one set a corresponding element of a second set. For example, by using Social Security numbers, a function could be set up assigning to each working person in the United States a nine-digit number.

One set Piaget is concerned with is the set of structures by which scientists describe the world. The set includes Euclidean structures such as straight lines, triangles, and rulers, which are used to describe spatial qualities; it includes logical structures, such as implication and negation, used to describe propositions; it also includes moral structures, such as obligation and fault, used to describe social relations.

The second set Piaget is concerned with is the set of a given person's structures of knowledge. Piaget's studies of children can be understood as attempts to discover to what structures of knowledge given scientific structures correspond.

For example, consider weight. As scientists consider weight, the weight of a given object is judged to have increased if something has been added to the object, to decrease if something has been taken from it, and not to have changed otherwise. But to the five-year-old child, an object can decrease in weight if its shape is changed so that it is shorter, even if this entailed no removal of matter from the object. The five-year-old assimilates weight to a different structure from the one scientists use.

Young children, in their drawings, represent triangles, squares, and circles all as scribbled closed figures. If they are blindfolded and given plastic figures, they cannot distinguish among triangles, squares, and circles, though they can distinguish any of these from a doughnut shaped figure. That is, the figures which scientists would describe in Euclidean terms such as triangles, squares, etc., the child assimilates to topological structures such as closed or open figures.

Consider a final example, taken not from Piaget but from an anthropologist, Laura Bohannon (1967). Bohannon attempted to tell the story of Hamlet to a group of African tribesmen in their native language, Tiv, in which she was fairly fluent. But the story that was heard was entirely different from the story that was told. For example, a focal point of the story is that Hamlet's uncle married Hamlet's mother shortly after Hamlet's father was murdered. To English speakers, the marriage was a suspicious act which automatically leads to speculation of wrong-doing on the part of Hamlet's uncle. To the Africans in question, the marriage was an honorable act as it is a man's obligation to marry his brother's widows. Both languages have a word for "father's brother", but the related concepts are differently structured. One entails an expectation that a "father's brother" may become a step father, the other entails a definite expectation that this will not occur.

A summary of uses of structures of knowledge: Piaget uses structures of knowledge to describe the constructive aspect of knowledge. Piaget needs a means of describing a person's knowledge which will allow a comparison between the properties of a part of a person's environment and the properties of the person's knowledge of his environment. This comparison can be made by using the concept of structure or schema.

Further, studying structures of knowledge can help us understand how a person constructs his world. An awareness of the extent to which these constructions vary between individuals and between cultures is the key to predicting and preventing misunderstanding.

Although it is useful to study the structures of knowledge it is not necessary. We now consider a theory which deliberately ignores the concept of schema.

Behaviorism has had such a major impact upon modern psychology that we assume the reader is acquainted with it, and with the position of B.F. Skinner.

Skinner's Position. Skinner considers statements about mental structures to be unnecessary at best, and a distracting metaphor at worst. Skinner is convinced that mental terms can be reduced to descriptions of behaviors, their antecedents, and their consequences. He finds the study of mental constructs to be useless. In About Behaviorism (Skinner, 1974) he dismisses Piaget in one

sentence, and portrays the history of psychology as a succession of dreary failures until behaviorists learned to restrict their attention to the study of environmental contingencies and physiology. The results of this restricted attention have indeed been dramatic. With the behaviorist approach it is possible to predict and control many complex human activities. But it is incorrect to conclude from this success that behaviorism has more scientific validity than do other schools of psychology that employ hypothetical constructs in their theories.

Behaviorism as a philosophy: Skinner's proscription against studying mental constructs must be seen as philosophical rather than scientific. Behaviorism is a philosophy, not a science (Strike, 1974; Skinner, 1974, p. 3). When a behaviorist states that what is called "thinking" is behavior, he is not stating the result of a scientific investigation of the phenomenon of thinking which has revealed that there is nothing but behavior involved in the phenomenon; rather, he is prescribing a way of talking about thinking which excludes mentalistic language. Similarly, when Skinner (1974, p. 127) says "A child learns to catch a ball without 'in some sense extracting the rules governing trajectories,'" he is not making an empirical statement. Skinner is probably aware of data which lead Piaget to

conclude that even preverbal infants follow rules that they "somehow abstract" from experience. Skinner's disagreement is not with Piaget's data but with the way he draws his conclusions; it is a philosophical disagreement.

In effect, Skinner's view is this: Instead of describing the structures of a person's knowledge, one should study the responses the person would have to given stimuli. And instead of explaining a person's behavior by referring to the structures of his knowledge, we should seek explanation in the person's reinforcement history. Skinner would rather scientists talk only about observable events.

For reasons too complex to discuss here, philosophies of science which forbid talk of anything but observable events have proved inadequate.² In biology, for example, the chromosome is observable but the gene is not. Yet, as the Lysenko affair proved, an adequate theory of genetics requires incorporation of the gene as a hypothetical construct. In physics an adequate theory of the electron requires a construct called spin which is both hypothetical and unimaginable in terms of normal experience. Strike and Wittgenstein state that there is no way to replace statements about structures of knowledge except by reference to those structures. (See Strike, 1974, pp. 111-112; Wittgenstein, 1953, pp. 31 ff.). Thus, Skinner's belief that the study of structures of knowledge is unnecessary is based on a questionable philosophy of science.

Summary: Why Study the Structures of Knowledge?

Piaget has several reasons for centering his studies on structures of knowledge. In addition, it seems that some psychologists might benefit by focusing more on these structures. The major points discussed are:

1. Piaget does not consider knowledge to be a passive and exact copy of reality. Rather, he considers knowledge to be a construction based on one's structures of knowledge. The notion of construction is needed to explain why in knowing things we systematically attribute to them properties they do not have. In order to study human knowledge, then, it is not sufficient to study the properties of what is known; the structures by which it is known must also be studied.

2. Piaget is interested in how knowledge becomes more valid. He sees the scientists's and the adult's knowledge as being more than a matter of the child's knowing less than the adult does. He sees the child's concepts, the structures of the child's knowledge, as being less valid than the adult's. He thus focuses on the structures of knowledge to see how they become more valid.

3. Skinner has raised the objection that one need not study structures of knowledge, nor any other mentalistic constructs. Rather, he believes it is sufficient to study the reinforcement history that has led a given person to act in a given way.

But there are patterns of behavior which can be described or explained only in terms of structures of knowledge. For example, the listener's response to "Hamlet" depends on whether one has a European or a Tiv concept of "uncle". Knowing the listener's concept, we can predict his reaction to the story. In order to account for the listener's reaction without referring to structures of knowledge, we would have to specify the necessary and sufficient conditions of reinforcement history which would produce a given concept of "uncle". On philosophical grounds, it appears impossible to give such a specification, and structures of knowledge must then be referred to to explain or predict the listener's reaction.

Why Study the Genesis of Structures of Knowledge?

Not all theories that focus on structures of knowledge are concerned with the genesis of those structures. But to Piaget the development of structures is integral to the structures themselves. Piaget's interest in development (genetic epistemology) has many roots.

Because it is there. A motive that probably underlies all investigations into this area is simple scientific curiosity. It is obvious that we acquire concepts and that our concepts change. The development of our concepts presents itself as a phenomenon to be explained.

This curiosity does not engage every investigator. Clark (1969) for example, is interested in the transitivity in adult reasoning, but appears uninterested in how that transitivity developed. Also, some people's curiosities are more easily satisfied than others'. Anderson and Bower, for example, recognized a need for their model of human associative memory (HAM) to be able to acquire new concepts. The need was filled, however, by a rule that states,

A universal concept is introduced into HAM as a consequence of encoding a propositional input that has one of its nonmemory nodes linked to a memory node by a subset (\subseteq) relation. When this proposition is encoded, this new memory node will represent a new universal concept. Such propositions we will refer to as concept-defining propositions. (1973, p. 191).

This states little more than that concepts are acquired. It does not explain how concepts are acquired, in the sense that it does not explain the origin of concept defining propositions. Also, it does not explain how concepts change after they are acquired. (This, of course, does not detract from the validity of the rest of the model. A model maker need not be able to answer every question he raises.)

But Piaget and several other theorists discussed below have been driven to study in greater detail how the structures of our knowledge develop.

Because the structures are constantly developing. Some theorists (Skinner, Anderson & Bower) see the growth of knowledge as the accumulation of facts (or responses) within a fairly stable structure. For Piaget, however, our knowledge is constantly being restructured.

For many philosophers and epistemologists, epistemology is the study of knowledge as it exists at the present moment; it is the analysis of knowledge for its own sake and within its own framework without regard for its development But . . . scientific knowledge is in perpetual evolution: it finds itself changed from one day to the next. As a result, we could not say that on the one hand there is the history of knowledge, and on the other its current state today, as if its current state were somehow definitive or even stable. The current state of knowledge is a moment in history, changing just as rapidly as the state of knowledge in the past ever has changed. (Piaget, 1970, pp. 1-2).

To account for the increase in validity. Piaget sees not only constant change in the structures of knowledge, but he also sees a fairly constant direction to that change viz., toward greater validity. Knowledge within a phylum, a person, or a science, tends to become more adequate, and this increase in adequacy is not only quantitative (how much is known) but also qualitative (how well structured is the knowledge). Piaget is therefore interested in the genesis of the structures of knowledge to account for the prevailing direction of this genesis.

For educational purposes. Education consists not only of increasing students' knowledge but also of changing the structures of students' knowledge. Educators therefore have an interest in the genesis of structures of knowledge.

Issues in the study of development. For those who are interested in the development of the structures of knowledge there are two central issues.

First, which structures of knowledge are innate? Obviously, not all of the structures are innate; people in different cultures have somewhat different concepts. On the other hand, all theories seem to posit some structure or structures as being innate. As to which structures are innate, there is a variety of answers from Piaget's schemes (ie. it is innate that ones knowledge is organized by schemes) to the Gestaltist's notion of good forms.

The second question is how new concepts are acquired and how old ones change. One view is that new structures are taken from structures of other areas: language, perception, or the world. Piaget's position is that new structures are constructed through the use of old structures.

ALTERNATIVE THEORIES

We are accustomed to look to genetics and the environment for explanations of psychological phenomena. But Piaget's notion of construction is somewhat akin to creation; when we know, we go beyond what is given to us by genetics or the environment. To see why Piaget feels a need to take this route, we should examine a range of theories which locate the origins of the structures of knowledge more directly in heredity and the environment.

Five theories will be considered. B.F. Skinner's and Eleanor Gibson's hold that the structures of knowledge are taken from the structures of the world. Benjamin Whorf's and that of the logical positivists see the structures of knowledge as coming largely from language. And the gestaltists see knowledge as being like perception and structured by genetically given mechanisms.

Gibson and Skinner: Structures of Knowledge taken from Structures of the World.

Eleanor Gibson and B.F. Skinner both believe the structures of knowledge to be taken from the structures of the world. They differ, however, in how they believe the structures are acquired, and in their attitudes towards structures of knowledge. Gibson is directly concerned with how concepts are acquired, while Skinner, as we have seen, considers all mentalistic terms to be distracting metaphors.

(It is thus only metaphorically that any position on the origin of the structures of knowledge can be attributed to him.) Gibson's theory will be considered first.

The basics of Gibson's theory. Gibson's theory consists of a metaphysics, a hypothesis about what is learned in perceptual learning, and a hypothesis about how that learning takes place. I believe Gibson's theory differs from Piaget's on all three points.

Gibson's metaphysical position is that the world has a structure. Her hypothesis about what is learned in perceptual learning is that we learn to detect that structure and its critical features.

I assume that there is structure in the world and structure in the stimulus and that it is the structure in the stimulus . . . that constitutes information about the world. That there is structure in the world is self-evident to the physical scientist who . . . discovers it. (1969, pp. 13-14).

What is learned (in perceptual learning) can be described as detection of properties, patterns and distinctive features. (1969, p. 77).

Gibson cites three mechanisms of learning: abstraction of critical features, learning to filter out irrelevant stimuli, and learning to attend to critical properties.

An alternative to Gibson's metaphysics. In defending her own metaphysics, Gibson contrasted it with only one other: the view that the world is unstructured and that the meaning-

fulness of stimuli depends on "information derived from other sources than the stimulus." (1969, p. 75). I would suggest that there is at least one other alternative: the world is infinitely structurable.

Consider as simple a figure as a red square. What is its structure? How is the figure correctly conceptualized? Depending on the context and the viewer, the figure could be structured as:

- "square" + "red"
- a red rectangle cut in half
- two red triangles put together
- a symbol of communism
- an orange square minus a yellow square

That is, it is not a matter of discovering the structure of a thing (or the critical features of that structure) as, if anything, choosing among its structures (see Wittgenstein, 1953, pp.22-23).

On Gibson's view, learning is a matter of taking the covers off of what was there all along and just seeing it. Of scientists who made a major breakthrough in the study of a particular enzyme, she wrote:

The scientists who discovered this complex structure were indeed performing a job of intellectual construction involving more than perception, but the structure was there to be discovered, not purely imaginary. (1969, p. 14).

On the learning of conservation of volume, she wrote:

Piaget has taught us to think of conservation as an intellectual achievement, and in its more conceptual modes it surely is . . . But I should maintain that perceiving the event of the water being poured and perceiving the sameness of the

water while it flows through a shape transformation from one vessel to another are fundamental to acquiring the concept of volume, rather than the other way round. (1969, p. 388).

To extend this reasoning, would we say that the non-Euclidean structure of the universe was there to be found by Einstein? Would we then also have to say that the Euclidean nature of the world was there to be discovered by Newton? Or that the change in volume of water when it is poured is there to be discovered by the preoperational child?

Gibson's view that structures of knowledge are actually structures of the world is incoherent because people systematically have differently structured knowledge of the same aspect of the world.

Gibson's view of what is learned. Even assuming there is a structure to the world, there is room to doubt Gibson's view that what is learned is detection of critical properties of that structure. A comparison of earlier to later forms of knowledge (child knowledge vs. adult knowledge, adult knowledge vs. scientific knowledge, earlier science vs. later science) generally shows inadequacies in the earlier form of knowledge which can be attributed to faults in the structure of the earlier knowledge. (It is not that Einstein knew more than Newton, but that the non-Euclidean structure of his physics is more productive than the Euclidean structure of Newton's physics.) At no point in time do we have grounds for believing that our current knowledge has the same structure as the world; it seems safer to assume that later developments will show some inadequacy in the current state of knowledge.

(see Kuhn, 1962.)
It seems doubtful, then, that what is learned at any time is, exactly, properties of the structure of the world.

It was with the systematic error in knowledge in mind that Piaget undertook his study of perception. He found optical illusions -- systematic distortions and over- and under- estimations -- to be the rule rather than the exception, and he endeavored to account for the more or less adequate knowledge we have through perception. In Principles of Perceptual Learning and Development (1969), Gibson devotes three pages to Piaget's studies of perception, and she states that few scientists have found Piaget's work to be interesting or useful. What is interesting is **here is her own dismissal of Piaget's approach:**

I am left with a stronger impression than ever that the geometric illusions are a far cry from perception of real things and pictures of real things, and that one should be cautious in generalizing from them to development of perception of ecologically valid sources of stimulation. (1969, p. 409).

To begin with, Gibson's position that line drawings are not "real things" is questionable. After all, it is not the drawing itself that is illusory but rather the properties we tend to attribute to it. More important, it is not just in line drawings but also in the wider world that we systematically over- and under-estimate. (Similarly, it is not just in the laboratory that the child misjudges volume.)

The position that line drawings are not "ecologically valid" sources of knowledge is also questionable. Clearly, our perception-based-knowledge of lengths in the Müller-Lyer illusion is geometrically invalid, but there may be ecological

47.

validity to the mechanisms producing the distortion. Hans Furth (1969) has speculated on this possibility:

Biologically speaking, the fact that the sensorial focussing brings about a relative overstimulation makes sense. Think of the apparant amplitude of a warning signal that must be noticed against background noises. Is this perhaps an indication that our sense organs have not evolved in order to bring objective knowledge, but in order to provide knowledge that is useful for the adaptive functioning of the organism? (p. 139).

One more point may serve to clarify this issue. Piaget, having decided that knowledge is a construction, is particularly interested in cases where knowledge is systematically invalid. In such cases, one can observe the difference between knowledge (e.g. that line A is longer than line B) and reality (that lines A and B are equal) and study what it is the knower does to reality in knowing it. On this view, cases where knowledge tends to be fairly valid are of less interest. The validity is seen as the result of compensations of errors, and the errors must be understood first.

Gibson, however, sees knowledge as being more like direct and undistorted contact with the environment. On this view, one would be more interested in studying valid knowledge and its development; optical illusions would be seen as figures that fool us, and would be only of peripheral interest. As the study of errors is of central importance in Piaget's approach and of peripheral relevance to Gibson's, it might be expected that she would not find his study of illusions to be valuable.

Gibson's mechanisms of learning. As was mentioned above, Gibson hypothesizes three mechanisms in perceptual learning. First, she says, a concept can be learned by learning to distinguish its critical features:

I believe the process by which the critical dimension is discovered is abstraction. . . . Abstraction occurs when an invariant relation is discovered over a number of varying objects or events. (1969, p. 108).

Her second mechanism is reciprocal to abstraction, and is the filtering out of irrelevant stimuli:

Idiosyncratic variations are ignored. . . . (The role of ignoring noncritical stimulation) is shown in experiments with imbedded figures, where practice seems to lower the threshold for seeing the contour of a design despite camouflaging lines present in the display. (1969, p. 111).

The third mechanism is learning to attend to critical properties. What is learned here is how to block out irrelevant stimuli from the nervous system. This involves such actions as turning the head toward a sound, focussing the eyes on a particular part of the visual space, and directing the fingers toward contact with a particular part of an object. All of these actions serve to minimize entry into the nervous system of irrelevant stimuli.

One obvious criticism of these mechanisms is that they do not explain much, but rather serve to name that which must be explained. Gibson's description of abstraction leads to the question, "How are critical properties abstracted?"

Similarly, one would want to know how a person "learns to attend to critical properties" and how one learns to ignore irrelevant stimuli. Of itself, this is not a serious criticism. If Gibson only rules out alternative mechanisms of learning and raises the above questions, she makes an important contribution to psychology.

I believe, however, that she is raising the wrong questions. To begin with, there is good philosophical reason to doubt that most concepts or even classes of objects have critical features. (Wittgenstein, 1953, pp. 31 ff.) Gibson tends to use examples where there are critical features. For example, in one experiment, she presented subjects with a collection of spirals which differed from a target spiral in size, number of curls, and/or direction of turning. (Gibson, 1969, pp. 77ff). The tendency for a given spiral to be mistaken for the target was shown to be a function of the number of dimensions by which it differed from the target. Also, subjects tended to confuse fewer spirals with the target after practice. Gibson's conclusion was that subjects learned to accurately recognize the target spiral by identifying the three critical dimensions.

Gibson does not explain, however, how one learns to identify size, number, or direction of curl in the first place; what are their critical features? She also does not concern herself with cases in which critical features are not evident, if existent at all. What critical features

define tables? Or the body type or face type characteristic of a given family? Or the continuity of appearance or personality of a person as he grows up? It has been argued that in these instances there is no common thread running through the concept -- no critical features which capture the concept. Without countering such an argument, Gibson's mechanisms seem inadequate to explain the learning of some common and important concepts.

Finally, if, as has been argued above, Gibson's hypothesis about what is learned in perceptual learning is wrong, her hypotheses about how such learning takes place would also be mistaken. If perceptual learning does not consist of learning to detect properties and patterns of the world, then perceptual learning cannot be explained as the result of abstracting and learning to attend only to critical features of such properties and patterns.

Summary on Gibson. Gibson's theory entails a metaphysical position, a hypothesis about what is learned in perceptual learning, and hypotheses about how that learning takes place. Arguments have been given against all three parts of the theory.

Gibson proposes a world with a structure. The alternative offered here has been a world which is infinitely structurable. It has been argued that any phenomenon can be validly construed not in one way but in an infinite variety of ways.

Gibson's theory holds that what is learned in perceptual learning is the detection of critical properties of the world's structure. It has been argued here that what is learned is often systematically different from what might be called the structure of the world or its critical features.

Finally, Gibson proposes that one learns by abstracting critical features of objects and concepts and by learning to attend to these features and exclude irrelevant stimuli. It has been argued that these mechanisms could not account for the learning of common concepts which have no critical features. It has also been noted that these mechanisms are in need of explanations themselves, and that their cogency depends upon the validity of Gibson's metaphysics and her hypotheses about what is learned in perceptual learning.

B.F. Skinner's position.

B.F. Skinner talks of the structure of knowledge only as a metaphor, but his view of it, like Gibson's, is that the structure of knowledge is copied from the structure of the world.

Not only is verbal behavior said to show the operation of innate rules of grammar, but 'innate ideas such as shape, size, motion, position, number and duration' are said to give form and meaning to the confused fragmentary data that we experience every day in our lives. Size, shape, motion, position, number, and duration are features of the environment. They have prevailed long enough and behavior with respect to them has been crucial enough to make the evolution of appropriate behavior possible, but

contingencies of reinforcement are at work every day in the life of the individual to generate supplementary behavior under the control of the same features. (1974, p. 116).

Unlike Gibson, Skinner sees no explanatory value in mentalistic constructs such as ideas and concepts (whether innate or not). He would rather study the behaviors which evidence concepts, and the environmental conditions which might produce behaviors and patterns of behaviors. He would rather study structures of behavior than structures of knowledge, and he sees the environment as determining the former: "It may be that there is no structure without construction, but we must look to the constructing environment, not to a constructing mind." (1974, p. 117). The way the environment structures behavior, according to Skinner, is given by the law of effect: "One effect of a successful behavior is to increase the probability that it will happen again in similar circumstances." (Rachlin, 1970, p. 73).

On the adequacy of Skinner's environmentalist explanation.

Let us examine Skinner's explanation in terms of an example, say, teaching a cat not to scratch furniture. The training is accomplished by hitting the cat whenever it scratches any piece of furniture. The punishment has the effect of reducing the probability that the cat will emit the same response to the same stimulus.

What I see missing in the behavioristic explanation of this learning is an adequate accounting for the word "same" as used in the phrases "same response" and "same stimulus". The trainer knows what the relevant stimulus objects and responses are, but how does it happen that the cat comes to refrain from just those responses to just those stimuli? How can a finite number of punishments prevent an infinite variety of responses to an infinite variety of stimuli? (The cat has learned not to scratch in any way any part of any furniture.)

Supposedly the only property common to all instances when the cat was punished -- but missing from all instances when the cat was not punished -- was that the cat had just been scratching furniture. It is this commonality that defines scratching furniture as a behavior to avoid, according to behaviorism. What I would want to argue is that the instances in which the cat was hit had numerous things in common which are missing from instances when the cat was not punished.

They are all instances when the cat is scratching furniture in the presence of the trainer (though the cat will usually generalize and learn not to scratch furniture even in the absence of the trainer). They may all be instances when the cat scratches a spot fourteen inches above the ground

(though the cat might later scratch at that height on its scratching post, but might never scratch furniture at any height). They may all be instances when the cat is leaning at an angle of 65° - 75° (though the cat learns not to scratch while leaning at any angle and may later lean against the wall at 70°). They might even be the only instances in which the cat scratches while music is playing.

The point of these examples is that no finite set of examples can contain all the information needed to properly generalize from the set. A purely environmental explanation cannot show why superstitious learning is the exception rather than the rule. The environment cannot define for the cat what "scratching furniture" is; the cat must make a contribution to the learning. Once the cat knows it is being punished whenever it scratches furniture, it makes perfect sense for it to mend its ways. What Skinner has not accounted for is how the learning takes place, how the organism construes the "lesson" in just the right way³.

Whorf and the Logical Positivists: Structures of Knowledge
Taken from Structures of Language.

There is a strong common sense appeal to the notion that structures of knowledge derive from words and grammatical structures. It is clear that to every word there corresponds at least one structure of knowledge (or scheme or concept). Also, some of our concepts relate to grammatical categories

and rules. We break the world into actions and objects (verbs and nouns), and we break time into past, present, and future (though speakers of languages with different tense structures from ours break up time differently). These common sense notions about the relations between structures of language and structures of thought have been formalized into theories by the logical positivists and by Benjamin Lee Whorf.

Whorf's position. Whorf's writing is full of examples of the relation between thought and language. One gets the impression that he believed that a person's language determines the way he knows the world. However, he tended to shy away from statements about causality, while speculating and citing examples of how speakers of different languages would have different ways of knowing the world.

Whorf's cross-cultural examples relate both to semantics and to grammar. As an example of the former, he pointed out that Eskimos have many words for the English "snow" and "slush" (including words for "snow to build igloos with", "snow that is good for sledding" etc.), while Aztec has only one word for "cold", "ice", and "snow". Eskimos know snow differently from the way we know it. They know it not just as snow, but as a particular kind of snow with a particular use. Again, one gets the impression that Whorf believed that it is because Eskimos have a richer snow vocabulary that they know snow in a more discriminating manner.

Whorf compared English tense structure with that of Hopi and explained why English speakers keep records and histories while Hopi speakers do not on the basis of this comparison. The English language treats time as a dimension like length which can be broken into equal units and which consists of past, present, and future. In contrast, Whorf writes:

(The Hopi view of time) is too subtle, complex, and ever-developing, supplying no ready-made answer to the question of when "one" event ends and "another" begins. When it is implicit that everything that ever happened still is, but is in a necessarily different form from what record or memory reports, there is less incentive to study the past. As for the present, the incentive would be not to record it but to treat it as "preparing". But our objectified time puts before imagination something like a ribbon or scroll marked off into equal blank spaces suggesting that each be filled with an entry. (1956, p. 153).

Whorf gave another set of examples from his days as an investigator for a fire insurance company. In one instance, workers who were very careful around full gasoline drums, were careless around "empty" gasoline drums (which were full of gasoline vapor which is much more flammable than is gasoline). He explained the workers' behavior as being a result of a confusion of "empty", meaning "containing nothing except vapor, liquid vestiges, or stray rubbish", with "empty" meaning "null and void, negative, inert".

In another factory, workers allowed a substance called "spun

limestone" to become overheated; the substance changed chemically from the heat, became flammable, and caught fire. "Behavior that tolerated fire close to the (substance) was induced by use of the name 'limestone', which because it ends in '-stone' implies non-combustability." (1956, p. 136).

Whorf's summary of his fire insurance examples might serve as a general statement of his position on the relationships among language, thought, and reality:

Such examples, which could be greatly multiplied, will suffice to show how the cue to a certain line of behavior is often given by the analogies of the linguistic formula in which the situation is often spoken of, and by which to some degree it is analyzed, classified, and allotted its place in that world which is (to quote Sapir) 'to a large extent unconsciously built up on the language habits of the group.' And we always assume that the linguistic analysis made by our group reflects reality better than it does.

Note that Whorf, like Piaget, focuses on differences between reality and properties we attribute to the world when we know it. Unlike Piaget, however, he sees language as the medium through which we filter the world.

The logical positivists. The logical positivists attributed a somewhat narrower role than did Whorf in explaining human thought and knowledge. The positivists held that logic is "only a general syntax in the linguistic sense." (Piaget, 1973). That is, logical structures are in essence linguistic

structures and are learned by learning to speak. As Piaget considers logical structures to be among the more important structures of knowledge, he has taken some pains to refute the positivists' position. He presents their position as follows:

The principal operatory structures (of logic) are, it is true, included in current language in a form which is either syntactical or inherent to the meanings (semantic). . . . Keeping to the current meaning of the words sparrow, bird, animal, and living being, the subject speaking can conclude that all sparrows are birds, that all birds are animals, and that all animals are living beings without the reciprocal being true, which constitutes a heirarchical interlocking of categories, that is, a classification. To state, on the other hand, that whales are both mammals and aquatic animals consists in expressing an intersection . . . The comparitives 'greater than,' etc., lead to seriations, etc., and the series of whole numbers are part of the current vocabulary. As for propositional or formal operations, language formulates the principal ones: the implication ('if . . . therefore'), the exclusive or nonexclusive disjunction ('either . . . or'). . . . As for structures much too differentiated and refined to be expressed by current language, mathematicians and logicians created for their own use artificial or technical languages but which, psychologically, are still languages. (1973, pp. 110-111).

Again, on this view, logic would be learned by learning to speak correctly. For behaviorists who adopted the positivist position, language could be learned through reinforcement of correct speech. One result of this position on logic is that what Piaget would call the consistency or self-regulated character of knowledge could be explained without recourse to mentalistic terms. The tendency for a

person's knowledge to be self-consistent could be reduced to a consistency of verbal reports, which could be reduced to correct use of language. This is contrary to Piaget's position that consistency of knowledge is the result of internal processes, and that logic is built by coordination of actions.

Piaget on the role of language in knowledge. To my knowledge, Piaget has not commented on Whorf's position. His remarks on the logical positivists, however, apply equally well to Whorf. His major argument is that there is logic in the infant's action, and structure in the infant's knowledge before language has been learned.

Indeed, a scheme is what is generalizable in a given action: for example, after having attained a distant object by pulling the blanket on which it had been placed, the child will generalize this discovery into using many other aids to draw closer many other objects in various situations. . . . In generalizing themselves, the schemes first constitute kinds of classifications. (1973, p. 114).

In addition to classification, the logical notion of seriation is demonstrated in the infant's action in his deliberate ordering of a series of actions in order to attain a given goal. The logical structure of a group is demonstrated in the infant's maneuvering around detours. (The infant learns that moving from point A to point B, and then from point B to point C, brings the same result as going directly from point A to point C; he also learns

that moving from point A to point B, and then from point B to point A, brings the same result as staying at point A.)

As a final example:

The coordination of schemes leads, moreover, to practical inferences: Seeking an object beneath a cloth under which a beret had been placed and not seeing the object when he raises the cloth, the infant sixteen to eighteen months old at once concludes that the object is beneath the beret, since this object had been slipped beneath the the cloth and that in raising the cloth he fails to see it. (1973, p. 115).

Classification, seriation, the notion of a group, and inference are not present at birth. They must be learned, and they are learned without benefit of language. Rather, Piaget finds in them the foundations of the same logical structures that are later apparant in speech. And the concept of the object, and concepts (schemes) for various objects predate (and enable) the learning of words for those same objects. Thus, both the positivists' and Whorf's positions seem untenable.

One other comment should be made about pre-verbal logic. It is easy to separate preverbal logic from the infant who exhibits it. After all, a stone, when it falls from point A to point B and then to point C, goes, in effect, from point A to point C. We do not, on account of this fact, attribute to the rock knowledge of groups.

The infant, however, learns to make use of logical structures in order to achieve his ends. When the infant detours around an obstacle he makes use of his knowledge that $AB + BC = AC$ (just as much as he makes use of his ability to crawl). Notions such as groups, implication, etc. structure the infant's knowledge, as do his schemes for "things that can be grasped", "things that can be sucked," etc.

How can language structure knowledge? If there is a correspondence between structures of knowledge and structures of language, how can the former be acquired from the latter? A language based explanation of the origin of structures of knowledge is incomplete without a description of a learning mechanism. But learning mechanisms are generally left out of language based explanations of the origins of structures of knowledge.

One mechanism that has been suggested in this respect is reinforcement. If logic is simply correct speech, supposedly it could be learned through operant techniques. I have already stated, however, why I do not feel operant principles can explain the acquisition of new concepts.

Another approach, taken by Chomsky, is to consider the rational core of knowledge and language to be hereditary. This is not the place to examine Piaget's views on Chomsky's rationalist position. (See Piaget, 1970, Section 16.) Basically, Piaget agrees with Chomsky's structuralism, but sees linguistic structures as the result of construction rather than of heredity.

Finally, there is a common sense notion, which Whorf might have had in mind, of how linguistic structures could determine structures of knowledge. Consider, for example, names of objects. (A similar argument could be made, with somewhat more difficulty, with other kinds of words and with other structures of language.) The infant hears the word "shoe", for example, in many different contexts; the contexts all have something to do with shoes, however. The word "shoe" in effect, serves to collect together for the infant a large set of shoes; it "tags" a large set of shoes as being, in a sense, all the same thing. The infant, on this account, would form a concept of "shoe" and would divide the world into shoes and non-shoes because of the use of the word "shoe" by its elders. An infant in a culture without the word "shoe" would not have the benefit of having shoes tagged in the same way as being similar, and thus would be less likely to form the corresponding concept.

This explanation is lacking in two respects. First, it begs the question of how concepts are learned. How does the infant abstract from the collection of shoes the notion of "shoeness"? Given that all the objects tagged "shoe"

have that attribute in common, how does the infant find out what it is? (Also, how does the infant come to recognize all spoken tokens of the word "shoe" as being of the same type?)

Second, this explanation, like the explanation by reinforcement, depends on there being something that all shoes have in common; it also assumes that the only thing situations in which the word "shoe" is used have in common is that they have something to do with shoes. As has been argued above, there is nothing all shoes have in common by virtue of which they are shoes; and situations in which the word "shoe" is used will probably have many things in common, not just some relevance to shoes. Therefore, exposure to correct use of the word "shoe" is not in itself sufficient to explain how one develops the concept "shoe". Such exposure might make it more likely that one will develop the concept "shoe" but some mechanism outside of language use is necessary to explain the development of the concept.

Summary on language based explanations. Two theories have been considered which claim that the structures of knowledge are taken from structures of language. Whorf held that the vocabulary and grammar of one's native language determine the structures of one's knowledge. He based his view on the facts that speakers of different languages know the world differently, and that because of the way we speak about certain situations we attribute to them properties they do not

have. The logical positivists held that logical structures (which Piaget considers to be important structures of knowledge) are simply linguistic structures, and that logical structures are learned by learning to speak correctly.

While it has not been claimed here that language plays no role in the development of concepts, two kinds of arguments have been given against the position that all concepts (or even all logical concepts) are learned through language. First, many concepts develop before speech develops; among preverbal concepts are concepts relating to objects ("bottle", "Mother", "pacifier", etc.) and logical concepts (seriation, groups, implication, etc.). Thus, many important structures of knowledge cannot be learned through language because they develop before language is acquired.

The second argument that has been given is that some mechanism other than language must be provided to explain how concepts develop; language itself does not constitute such a mechanism. It was argued that logic could not be learned through reinforcement of correct speech because reinforcement cannot account for the learning of concepts. It was also argued that language itself cannot cause concepts to be learned through indicating that a group of objects are all examples of a given concept; a mechanism would still be needed to explain how the concept is abstracted from the examples.

Gestalt Theory

Like Piaget's theory, gestalt theory holds that knowledge is not a copy of the world, but that we filter the world through structures of knowledge. Piaget summarized the gestaltists' position as follows:

It is well known that Gestalt theory considered the act of intelligence to consist in the restructuring of a given situation in the direction of a "better" form, the forms of intelligence obeying the same laws as those of perception. On this basis, intelligence would have to be thought of as a continuation of perception, whose structures would extend to elements outside the perceptual field. (1969, p. xiv)


While Piaget sees merit in the structuralist aspects of gestalt theory, he takes exception to three of its tenets: that the structures of perception are innate, that perception is structured by "good forms", and that perception and intelligence have the same structure. It is primarily with the last point that we need be concerned here.

The gestaltists' position. The gestaltists did much of their research in the area of perception. Their approach in this research was similar to Piaget's and different from Gibson's in that they saw optical illusions as being sources of information about the contribution made by the perceiving organism to its own perceptual knowledge. Koffka (1935), for example, commented on a tendency for psychologists to look for special conditions which mislead judgment when two equal lines appear to be unequal:

Illusory perceptions were not accorded the same rank as non-illusory ones; they presented a special problem, whereas the normal appearance presented no problem at all. This distinction between two kinds of perception, normal and illusory, disappears as a psychological distinction as soon as one becomes thoroughly aware of the fallacy which it implies, much as it may remain as an epistemological distinction. For each thing we have to ask the same question, "Why does it look as it does?" whether it looks "right" or "wrong"! (p. 79)

Even in cases where perception is accurate, Koffka rejected the solution that "things look as they look because they are what they are." (p. 77) He thus recognized the problem, ignored by Skinner, Gibson, and other reductionists, of how the organism structures sensory data. The gestaltists' solution to this problem is given in the Law of Prägnanz:

Psychological organisation will always be as "good" as the prevailing conditions allow. In this definition, the term "good" is undefined. It embraces such properties as regularity, simplicity, and others. (Koffka, 1935, p. 110).

The Law of Prägnanz would explain, for example, why a broken triangle, when seen briefly, would look like a complete triangle, or why we tend to see the following dots  as describing a square rather than two triangles or two lines or a triangle and a line; in the first case, a complete triangle has a better form, and in the second, the square is the simplest form appropriate. The Law also explains why triangles and squares are perceived without distortion; they are already good forms. It should be noted again that the gestaltists believed the laws defining "goodness" to be innate.

The gestaltists believed the law of Prägnanz to govern not only perception but also all intelligent behavior. To give just one example, Wertheimer tried to explain syllogistic reasoning in terms of the law:

"All men" constitutes a whole which is represented as located within the complex of "mortals". The minor term follows the same course; "Socrates" is an individual located within the circle of "men". So the operation which draws the conclusion from these premises, "therefore Socrates is mortal", simply amounts to restructuring the whole by abolishing the intermediate circle (men) after first placing it, with its content, within the large circle (mortals). Reasoning is thus a "re-anchoring". "Socrates" is, so to speak, uprooted from the class of "men" in order to be anchored in that of "mortals." The syllogism is thus without more ado related to the general organisation of structures. (Piaget, 1960, pp. 59-60).

Syllogistic reasoning is thus seen as a simplified restructuring of a perception.

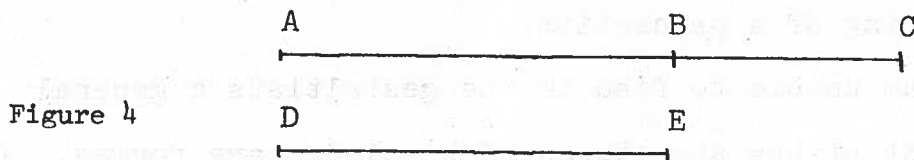
I am unable to find in the gestaltists a general statement of how structures of knowledge are formed. I do not know, for example, how they would explain the formation of classes such as "men", "mortals", etc. They do seem to indicate, however, that the knowledge we have of any given situation will have as "good" a structure as possible, and that all knowledge is structured according to the laws that structure perceptual knowledge.

Piaget on the gestaltists. Because the gestaltists see knowledge as being filtered through structures, their theories bear some fundamental resemblances to Piaget's. There are, however, three important differences relevant to this paper. Piaget does not believe perceptual knowledge to be structured

by the laws described by the gestaltists, he does not believe the laws of perception to be innate, and he does not believe all knowledge to be structured by the same laws as those which structure perceptual knowledge. The first two disagreements derive from Piaget's study of perception. An adequate treatment of the relevant data cannot be given here, and the interested reader is referred to Piaget, 1969.

Piaget (1969, Chapter VI) notes several differences in structure between perception and intelligence (higher order knowledge), but the most important is the property of reversibility. Perceptual structures are irreversible but structures of intelligence are reversible.

For example, consider Figure 4.



If the figure (without the letters) is presented tachistoscopically, segment AB will appear longer than DE, though they are actually equal. Thus, according to the laws of perception, $AB + BC - BC \neq AB$. The laws of perception do not allow deformations (eg. the addition of a line segment to a line) to be properly corrected; the initial conditions cannot be returned to.

Logical structures, on the other hand, are characterized by their reversibility. The child's concept of length attains

reversibility when he is around five years old; at that age he would know that $AC - AB = DE$. In logical structures, every deformation (eg. $\rightarrow BC$) has a corresponding compensation ($-BC$) which returns the situation to its original state. This reversibility, common to all of the conservations, as well as to classifications, seriations, groups of displacement, etc., is missing in perceptual knowledge.

Not all schemes are reversible; the preoperational concepts of length, weight, volume etc. are characterized by their irreversibility. The point here is that there is an essential characteristic to some structures of knowledge that is lacking in all structures of perception. No combination of irreversible structures will constitute a reversible structure. Of his studies of the differences between perceptual and intellectual structures, Piaget wrote:

We established, in the case of almost every one of the differences, that the transition from the structures of perception to those of intelligence required the intervention of contributions arising in actions or in operations, and that the transition could not be explained simply as an extension of or an increased mobility in, perceptual structures. (1969, p. 310)

The structures of perception, whatever they are and whatever their origin, cannot account for all of the structures of knowledge. Intelligence is not just a continuation of perception, and a different account must be given for the origin of the structures of knowledge.

A summary on Alternative Theories

A major purpose of this chapter was to show why Piaget needed a third factor, after genetics and the environment, to account for the development of structures of knowledge. To do this I examined the inadequacy of theories purporting to account for this development on the basis of genetics and environment alone, and in the case of each theory two kinds of arguments were made. First, each theory argued that all or some of the structures of knowledge are copied from structures given in the environment or by genetics; in each instance it was argued that the structures of knowledge must have a source different from that proposed by the theory in question. Second, most of the theories proposed mechanisms by which knowledge would acquire structures of other domains; in each case, the proposed mechanisms were considered to be inadequate to account for the acquisition.

The arguments given have been as follows:

1. Gibson argued that the structures of knowledge come from structures of reality; the mechanisms she proposed to explain how those structures are learned centered on the discovery of critical features of structures of reality. It was argued here that the structures of knowledge are often different from the structures of reality (if, indeed, reality has a set of structures). It was also argued that not all structures have "critical properties", so that structures of reality cannot be learned by discovering their critical properties.

2. Skinner argued that what are called structures of knowledge are just patterns in behavior deriving, through reinforcement, from the structures of the world. Again it was argued that the structures of knowledge are different from the structures of the world, so that the structures of knowledge must have a different source. It was also shown that the explanation of learning by reinforcement depends on there being just one thing in common to all instances when an organism is being trained for a given task; it was argued that any set of instances has an infinite variety of things in common, and that reinforcement alone cannot account for the organism's selecting out of the instances what the trainer has in mind for him to learn.

3. Whorf argued that structures of knowledge come from structures of language; the logical positivists argued that logical structures are essentially structures of language. It was pointed out that there is structure to pre-verbal children's knowledge, so that language cannot be the source of all structures of knowledge. It was also argued that no adequate account had been given of how structures of knowledge could be acquired from the structures of language.

4. The gestaltists saw hereditary structures of perception as the basis of all structures of knowledge. It was argued that the reversible structures of intelligence must have an origin outside of the irreversible structures of perception.

PIAGET'S EQUILIBRATION THEORY

So far in this paper I have discussed why one would be interested in the origin of structures of knowledge, I have sketched Piaget's theory of their origin, and I have shown why I believe other theories to be inadequate. In this final chapter it would be appropriate to explore the adequacy of Piaget's theory. One way to do this is to compare to the explanations given in the previous chapter a Piagetian explanation of the same phenomena.

The theories discussed in the previous chapter raise four questions: How is it that our knowledge of the world is fairly valid? Why does reinforcement work? How is logic learned? What is the relation between knowledge and language? If theories built around these questions have failed to answer them adequately, we should explore whether Piaget's explanation is any more adequate.

Why Does Knowledge Tend to be Valid?

Piaget, as much as Gibson and Skinner, is aware of the adaptive value of knowledge. Whether reality has one structure or is infinitely structurable, the acquisition of new structures of knowledge (whether within an individual, a science, or the history of a species) tends toward greater validity. Piaget is definitely concerned with explaining the tendency for knowledge to become more valid.

Piaget's explanation. I believe there are six steps in Piaget's explanation of how structures of knowledge develop so as to increase the validity of knowledge.

1. Action. There are many kinds of action. Some examples might be grasping an object, judging the fitness of a given woman for a given job, and designing a system of government. All action entails assimilation to existing schemes; the object is assimilated as something graspable, the woman is assimilated as a woman and as a job applicant, the problem of designing a government is assimilated to various schemes relating to systems, needs and rights of governed people, etc. All action is in response to needs which are defined in terms of schemes; the infant needs to know his world and can do so by exercising his grasping scheme, the employer's need is defined in terms of his scheme for "employee" or "job applicant", and if one wants to design a government, his want is shaped by his concept (or scheme) of "government".

2. External inconsistency. Every scheme and every action has expectations; expectations are usually not met completely, and are sometimes found to have been quite inappropriate. The infant may find his fingers closing more than he had expected them to in grasping the object. The woman might be more competent than the employer had expected her to be given his scheme for "woman". The governmental system might not do exactly what it was designed to do; this could be discovered

by trying out the system and then hearing people's complaints, or it could be found out by thought and analysis leading to the realization of a problem.

3. Internal inconsistency. Once the external inconsistency becomes known, it becomes internal. The infant discovers that his hand did not stop closing in where he had expected it to; the motions of his hand are inconsistent with the expectations based on his schemes related to grasping. The employer discovers the woman to be doing a better job than he had expected; his knowledge of her performance is at odds with his expectation of that performance. There is an inconsistency in a system's not doing what it was designed to do. These inconsistencies lead to disequilibrium which is a need for equilibration.

4. Equilibration. The inconsistencies cause accommodations (changes in schemes) which may be great or small, permanent or temporary. The infant may just close his hand further than he had expected to. Conversely, he might differentiate his grasping scheme into two subschemes: one for grasping solid objects and one for grasping furry objects which have to be grasped tighter. The employer might just consider the woman to be unusually competent for a woman, or he might change his scheme for "woman" so that it does not imply incompetence. The planner might revise his planned government, and he might also change some of his ideas about government and systems. The changes in schemes will return the situation to a state of equilibrium.

5. Internal consistency. Equilibration is not the result of all kinds of changes in schemes, but only those which lead to greater internal consistency. The infant's grasping scheme would not change so as to lead to an expectation that the hand stays relatively open while grasping furry objects. Rather, the change leads to expecting the hand will close a lot when grasping furry objects; this new scheme is consistent with the fact that the infant's hand closed rather tightly around the last object it grasped. The employer's new concept of "woman" will be consistent with his knowledge of his new employee's competence. The planner's new concepts of government and systems will be consistent with his knowledge of the problems in his original plan.

6. External consistency. Before the schemes were changed, they were fairly consonant with earlier knowledge which was fairly valid. The new schemes will be consonant not only with earlier knowledge but also with the new discrepant knowledge. Accommodation should therefore lead to knowledge which is even more in accord with reality. In future, the infant will make a more accurate grasp for furry objects, the employer will be more open to believing a given woman to be competent, and the planner will plan governments without the flaws of his earlier models. It should be noted that the changes in schemes also implies a change in future actions. It is now a different thing for the infant to grasp, for the employer to interview a woman, for the planner to plan a government because the schemes for "grasp", "woman" and "government" have changed.

Theodore Mischel (1971) described the process of changing schemes as follows:

What leads the child to give up one of his (schemes) in favor of the next is that his cognitive assimilations keep running into difficulties because they conflict with "facts" available to the child, or with his other beliefs, or with what other people say, etc Thus, whatever resists assimilation to the child's schemas generates cognitive conflict, and the child's recognition of this "disequilibrium" motivates him to resolve the conflict: he accommodates his schemas in order to assimilate it. . . . The general motive, both for applying a schema one already has and for elaborating new schemas in the course of development, would be, in effect, the 'need' to make sense of present problems by fitting them coherently to schemas used in past solutions. (p. 331).

More simply, the reason changes in schemes leads to more valid knowledge is that schemes only change when an area of invalidity in an old scheme becomes apparent, and then only a change which will lead to greater validity will be adopted.

On the adequacy of Piaget's explanation. Piaget's explanation has two distinctive features. First, the same factor which initiates changes in schemes, equilibration, also determines what kinds of changes in schemes will become permanent. This gives accommodation a steady direction toward greater validity. Second, external consistency, the accord of thought with things, is achieved through internal consistency, the accord of thought with itself. This eliminates the circularity in, say, a behaviorist explanation in which the organism needs to know what he is being trained to do in order for the training to work.

What I am not sure of is whether Piaget's "equilibration" explains cognitive development or serves to name that which must be explained. Theodore Mischel seems to lean toward the latter interpretation:

Questions need answers and problems need solution for the person to whom they are problems or questions. But do we have to engage in empirical investigations in order to discover this? It is hard to see how someone could understand that something is a problem without understanding that it needs to be solved. . . . Could someone have the concept of "consistency" and yet fail to grasp that inconsistent beliefs need to be reconciled? . . . I want to suggest that Piaget's general account of equilibration, of the way in which the child's awareness of cognitive perturbations (conflicts) motivates his intellectual development and functioning does not constitute a theory to be confirmed or refuted by facts; it is an analysis, or rational reconstruction, of how we think in accordance with the norms that govern thinking - an analysis which Piaget uses as a framework for an empirical mapping of the stages through which the child passes in coming to think in accordance with the norms of adult logic. (197 , pp. 342-343).

I believe Mischel is saying, as Strike said of Skinner, that some of Piaget's theory is philosophy, not science; his interpretation of "equilibration" is that it is not something Piaget has "found" empirically but rather it is part of the language in which empirical findings should be described. On this understanding, there are two problems with equilibration that, to my knowledge, Piaget has not begun to solve.

First, the statement that schemes change in the direction of greater equilibrium is not sufficiently specific. There are many kinds of changes that could bring about greater equilibrium (just as there are many things any set of training

instances have in common). Cognitive conflict can be reduced by seeing an anomalous instance as being just an exception to the rule, or conversely, any number of changes could be made in any of several schemes which would also reduce disequilibrium. Piaget is very specific at times about the specific stages a given scheme goes through. The child judges the amount of a given substance, for example, first on the basis of height, then on the basis of width, and then, for a period, he will oscillate between the two. During this period of oscillation, the child begins to reason about and discover the interdependence of height and width. This leads to the final stage of the adult concept of amount. In general terms, Piaget has explained why each of these stages is successful: each brings about greater equilibrium. What is not explained is why those particular stages are always passed through. A move from judging by height directly to the stage of oscillation or to the final stage would also increase equilibrium. There are probably other changes that could, logically, be made in the concept of amount which would also increase equilibrium. Piaget's theory is not sufficiently exact to predict, in a given instance, how equilibrium will be reduced, nor can it explain why certain patterns of accommodation always hold.

The second problem with Piaget's explanation is that he does not explain how it is that a new scheme can be constructed at all, let alone a new scheme which will reduce disequilibrium. Other theories locate the origin of schemes in other areas; structures of the environment, structures of

language, or genetically given structures. If we take Piaget seriously, however, I believe he says that we create new schemes. We do not take them from any external source. They just happen to us. Again, the need to see changes in schemes as being the result of creation comes partly because Piaget has seen no adequate account of how the new structures might come from external sources. There is nevertheless something unsatisfying about an explanation by creation.

It should be noted that I do not see these two problems as reasons for abandoning Piaget's explanation. In particular, I see the need for further specification of the specific way in which equilibrium will be restored as being a direction for future research rather than a weakness in Piaget's theory. The explanation does rule out alternative explanations, and it calls for the discovery of laws to play the same role as the Law of Prägnanz did in gestalt theory. I am, however, puzzled by the absence of an explanation of how new schemes are created. This may be a philosophical rather than an empirical problem. The solution might not lie in finding new laws and mechanisms, but might rather lie in concluding that the creation of new schemes is in fact creation, not under control of environment and heredity, and therefore outside the realm of science.

Why Does Reinforcement Work?

Although Piaget has not, to my knowledge, tried to account for the effects of reinforcement training in his

terms, there is one point he has emphasized: the stimulus and the response are logically interdependent. What it means for something to be a stimulus is that it is assimilated to a scheme, and a scheme is a general way of acting on something. We never know things except as something to grasp, something to hire, something to scratch, etc. Thus, the connection between the stimulus and the response is not made by the reinforcement but rather exists to begin with in the fact that the stimulus is a stimulus; that it is a stimulus means, by definition, that it will facilitate a given response.

I believe reinforcement plays a different role in a Piagetian account of reinforcement training from the role it plays in a Skinnerian explanation. To Piaget, the trainer's application of reinforcement after a certain behavior is, first of all, a condition of the environment to be assimilated by the trainee. The trainee's schemes must change so that he now expects reinforcement to be a consequence of that behavior, and so that the behavior becomes a means to him of obtaining the reinforcement. With better knowledge of his environment, it makes perfect sense that the trainee is more likely to do that for which he is reinforced and avoid that for which he is punished. (This is true by definition of the words "reinforce" and "punish".)

Reinforcement affects learning in two ways. First, as was mentioned above, the trainee learns it is being reinforced for a certain action. Second, by making a certain action more probable, it accelerates the development of the

scheme directing that action. If, because of reinforcement, we are encouraged, say, to play with clay in a way which requires us to judge volumes of pieces of clay, our schemes for judging volume are likely to develop more rapidly. The learning is the result of accommodation caused by disequilibrium caused by action, and the action is made more likely because of the reinforcement. This is quite different from the Skinnerian explanation whereby the reinforcement serves to connect a response to a stimulus.

The Piagetian explanation of reinforcement training is much more cumbersome than the Skinnerian explanation, but it manages to account for the effects of reinforcement while avoiding the logical pitfalls inherent in the Skinnerian.

How is Logic Learned?

Some of Piaget's writings about logic are among his most difficult pieces to understand. He does indicate fairly clearly, however, two trends in the development of logic.

First, the development of any given scheme is in the direction of higher logical structure. Above all, this means the direction of greater reversibility; there become more ways in which some aspect of an object can change without the identity or quantity of the object changing. The concept of volume, for example, is at first irreversible; a change in an object's shape is taken to imply that its volume has changed. In the adult concept of volume, however, the change in shape can be mentally reversed, and the volume is understood to

remain unchanged. Similarly, the adult Euclidean concept of space is more differentiated and reversible than the infant's topological notions of space. Operations such as adding length to a line and changing the orientation of a shape are reversible in Euclidean geometry. The operations thus have a stability and a meaningfulness that is missing in the infant's geometry.

The second trend in the development of logic is towards greater consciousness and abstraction. For example, the infant may unintentionally group objects together, and may later group objects intentionally or for a specific purpose. At a later point he will be able to talk about grouping objects. The next development is for him to group objects together abstractly by doing addition problems. When he reaches the stage of formal operations he will be able to discuss the rules governing addition, and will be able to discuss even more abstract systems. The sensorimotor scheme of joining objects together thus is seen by Piaget as the origin of formal operational schemes of higher mathematics.

The cause of both of these trends is equilibration. Reversible structures are better equilibrated than irreversible structures. The adult concept of volume leads to fewer contradictions and is in better accord with the way things are than is the child's concept of volume. Greater abstraction and consciousness allows errors to be corrected before they are performed. Knowing how to do arithmetic can prevent a child from adding too many objects to a collection in order to

bring it to a certain desired size. Knowing the laws of arithmetic can prevent an adolescent from making a certain error in calculation. Knowing the laws of algebra can prevent a mathematician from making errors in constructing new mathematical structures. Thus, abstraction and consciousness lead to better equilibrated systems.

For Piaget, then, "logical structures result from the progressive equilibration of the pre-logical structures" (Piaget, 1967, p. 105). The roots of the highly abstract and reversible formal logical structures are the concrete and irreversible schemes of the infant. The motivating force of the development is equilibration.

What is the Relation between Knowledge and Language?

This question entails two issues: the origin of speech, and the correlation between structures of knowledge and structures of language.

The origin of speech. To Piaget, speech is enabled by the semiotic (or symbolical) function, i.e., the ability to represent one thing by another. Language is a system of symbols, meanings, or connections, and the roots of language are in the earliest schemes. To the infant, at various ages, sucking on the nipple means that milk will come, putting a handkerchief over a penny means the penny will be there when the handkerchief is removed, and pulling on a blanket means that whatever is on the blanket will come closer.

The semiotic function enters when the significant becomes differentiated from the signified. For example, although sucking on a nipple signifies to the infant that milk is going to come, the sucking and the milk are all part of a whole undifferentiated by the infant. On the other hand, a signifier is a symbol if it is clearly differentiated from the signified.

Piaget identifies three kinds of such signifiers: symbolical play (in which objects are represented by gestures), deferred imitation and drawing, and mental imagery. Each of these is done in the absence of what it represents.

Speech and other systems of social signs develop at the same time as the above three symbolic systems. To Piaget, speech is a system of symbols, a means of re-presenting what is not present. Knowledge must be fairly well developed for speech to be possible; to have a word for a given object, for example, the child must already have a scheme for the object which is developed to the point that he understands that the object exists apart from his actions on it. Piaget puts knowledge prior to language.

The correlation between structures of knowledge and structures of language. I believe that linguistic determinism is contrary to the Piagetian view. Structures of language do not determine structures of knowledge. The correlation between the two kinds of structures would be seen as the result of physical and social environments which influence both language and knowledge.

each was judged to be inadequate. In general, these alternative theories saw the structures of knowledge as arising from lower-ordered structures of other areas: reality, language, or perception. None of these theories provided a mechanism for developing a higher-ordered structure from a lower-ordered structure. For example, the gestaltists provided no means of developing reversible structures from irreversible ones, and neither Gibson nor Skinner provided a mechanism for constructing a concept given a collection of examples of it.

Piaget provides a theory in which structures of knowledge grow from previous structures of knowledge through action, disequilibrium, and equilibration. His theory gives an adequate accounting for the development of structures of knowledge while providing a mechanism, equilibration, to account for raising the level of structures. Further research is needed, however, for "equilibration" to become an explanatory rather than just a descriptive term.

NOTES

1. I use the word "knowledge" in the sense that psychologists do, meaning something like "belief". As philosophers use the word, knowledge is, by definition, valid, and a phrase like "valid knowledge" would be redundant. By "valid" I mean "true" or "in accord with the way the world is". As much as possible I would like to avoid in this paper the issue of how the world really is; for this paper, I will make the assumption that the description of the world given by scientists (in those fields where there is a consensus among scientists) is accurate. Piaget himself explores the notion of validity in some of his

(See Piaget, 1971).

2. Strike (1974) has done an excellent job of presenting this argument, and the interested reader is referred to his paper.

3. Gregory Bateson offers in one crisp example an indication of how much is left to be explained after the law of effect has been noted:

A certain mother habitually rewards her small son with ice cream after he eats his spinach. What additional information would you need to be able to predict whether the child will: a. Come to love or hate spinach; b. Love or hate ice cream, or c. Love or hate Mother? (1972, p. xvii.)

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